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AN ADAPTIVE FEEDFORWARD CONTROLLER FOR REJECTION OF PERIODIC DISTURBANCES

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The issue of rejecting periodic disturbances arises in various applications dealing with rotating machinery. A novel adaptive feedforward controller (AFC) design method for rejecting periodic disturbance is proposed. It can be added to an existing feedback control system without altering the original closed loop characteristics, such as the stability margin and transfer functions. An adaptive algorithm derived for the proposed controller based on the steepest gradient descent method turns out to be identical to the popular delayed-x LMS algorithm. A number of computer simulations were carried out to demonstrate the salient features of the proposed AFC compared to the conventional one. The proposed method can be useful for control engineers in designing an adaptive feedforward controller for the rejection of the periodic disturbances.

1. INTRODUCTION

Periodic disturbances occur in various control engineering applications, which can be represented as the block diagram shown in Figure 1. For example, the eccentricity of the track on a disk requires a periodic movement of the read/write head at the frequency of rotation of the disk [1]. Hence, compensators designed for regulation without considering the eccentricity cannot achieve asymptotic tracking of the actual track. The offset is periodic, with frequencies appearing at integer multiples of the fundamental harmonic (the rotating speed of the disk). This disturbance is particularly large if the disk is removable, such as in compact disc players. In [2], active vibration control is applied to a cryocooler expander to eliminate several harmonics originating from a rotating pump. The control of rotating magnetic bearing systems is another recent application of great interest [3, 4]. A problem common to all mechanical systems with rotating shafts, including the aforementioned cases, is synchronous vibration, a periodic disturbance, caused by mass unbalance: the condition in which the principal axis of inertia is not coincident with the axis of geometry.

Several methods are used to design linear control systems for eliminating periodic disturbances. One of the most common approaches is based on notch filtering, which adds a notch filter at the synchronous frequency (rotational speed) into the loop, as shown in Figure 2 [5, 6]. However, removing all signals at the synchronous frequency can result in closed-loop instability, limiting the usefulness of this approach. The other method is adaptive feedforward control (AFC) [4, 7, 8], in which the disturbance is cancelled by adding the negative of its value, $r_\Omega(k)$, as shown in Figure 3. AFC continually estimates the Fourier coefficients describing the sinusoidal disturbance and adapts to variations on-line. Since the conventional AFC is designed without considering the underlying structure of the system, it alters the original closed-loop characteristics.

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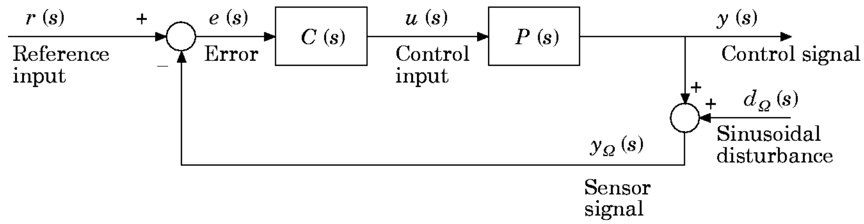

Figure 1. A block diagram of the feedback control system with periodic disturbance.

In this paper, a novel adaptive feedforward controller (AFC) design method for rejecting periodic disturbances is proposed, which avoids the drawbacks of the conventional ones. Contrary to the conventional AFC, it can be added to an existing feedback control system without altering the original closed loop configuration. Thus, the proposed AFC does not affect loop stability. We derive an adaptive algorithm for the proposed AFC design, which turns out to be identical to the delayed-x LMS algorithm [9], which can be considered as a special form of the filtered-x LMS algorithm [10]. The use of a more general filtered signal has been discussed by Morgan [11], Burgess [12] and Widrow *et al*. [13]. Snyder and Hansen [14], Morgan [11] and Long *et al*. [9] derived the bounds for the step size that makes the delayed-x LMS algorithm stable. The proposed AFC design and effectiveness of its algorithms are illustrated by simulations.

2. METHOD OF PERIODIC DISTURBANCE REJECTION

Consider the set-up in Figure 1, which shows a linear-time-invariant plant $P(s)$ perturbed by a periodic measurement disturbance $d_\Omega(t)$ represented as follows:

$$d_\Omega(t) = D \sin(\omega_0 t + \phi) = w_0 \sin(\omega_0 t) + w_1 \cos(\omega_0 t), \tag{1}$$

where $D = \sqrt{w_0^2 + w_1^2}$, $\phi = \tan^{-1} w_1/w_0$; w_0 and w_1 are the Fourier coefficients of the synchronous disturbance at the frequency ω_0. The sensor signal $y_\Omega(s)$ contains a synchronous measurement disturbance $d_\Omega(s)$. The compensator $C(s)$ is assumed to be designed to stabilize the system.

Several methods are used to design linear control systems for eliminating periodic disturbances. Let us briefly review the existing methods and their drawbacks.

Probably the most common approach is based on the use of a notch filter [5, 6], as shown in Figure 2. The notch filter used in this investigation is an ideal, two-pole transfer function with an infinite notch depth corresponding to the frequency of the disturbance. The notch

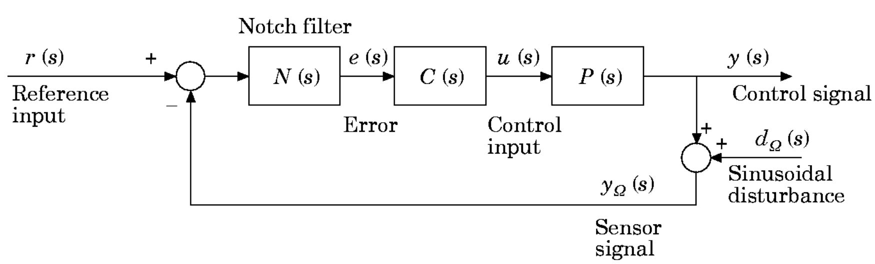

Figure 2. A block diagram of the feedback control using a notch filter for rejecting periodic disturbance.

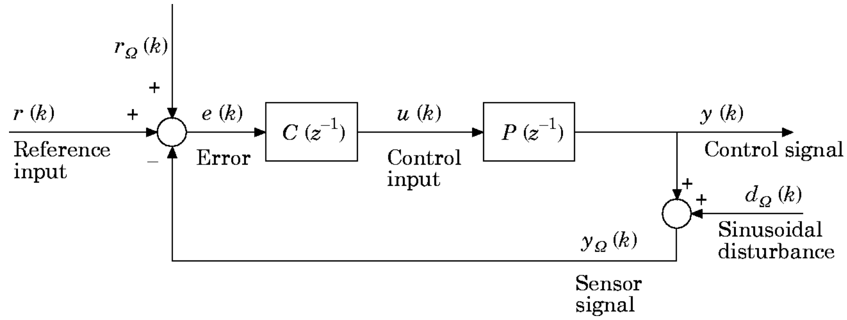

Figure 3. A block diagram of the feedback control for rejecting periodic disturbance.

transfer function, $N(s, \omega_0)$ is a function of both Laplace variable s and the disturbance frequency ω_0. It is given by

$$N(s, \omega_0) = (s^2 + \omega_0^2)/(s^2 + (\omega_0/Q)s + \omega_0^2). \quad (2)$$

The steepness of the notch is determined by the parameter Q. A large Q makes a steeper notch. The negative phase of the notch transfer function, $N(s,\omega_0)$ near the disturbance frequency may lead to instability, limiting the usefulness of this approach. The remaining design problem after adding the notch filter is to choose the transfer function $C(s)$ so that the closed loop transfer function is stable and has desirable input–output properties. The method is easily extended to the case in which the disturbance is the sum of two or more sinusoids. Poles are simply added at all frequencies of the disturbance. Note, however that, it becomes increasingly difficult to guarantee stability as poles are added.

Another method is based on the adaptive feedforward cancellation (AFC) shown in Figure 3. The disturbance is cancelled by adding the cancelling signal $r_\Omega(k)$, which has identical magnitude but opposite phase to that of the disturbance [4, 7, 8]. The cancelling signal has the form

$$r_\Omega(k) = w_0(k) \sin (k\omega_0 T) + w_1(k) \cos (k\omega_0 T) \quad (3)$$

where T is a sampling time and $w_0(k)$ and $w_1(k)$, $k = 0, 1, 2, \ldots$, are discrete time-varying Fourier coefficients updated on-line. Since the magnitude and the phase of the sinusoidal disturbance are generally unknown, they must be estimated. An adaptive algorithm can be used for that purpose. The AFC method is also easily extended to the case in which periodic disturbances with an arbitrary number of harmonics need to be cancelled. If the

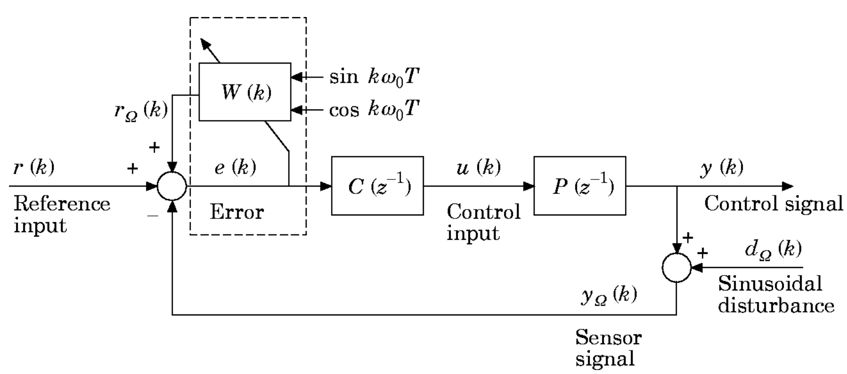

Figure 4. Block diagram of the feedforward control using an adaptive notch filter for rejecting periodic disturbance.

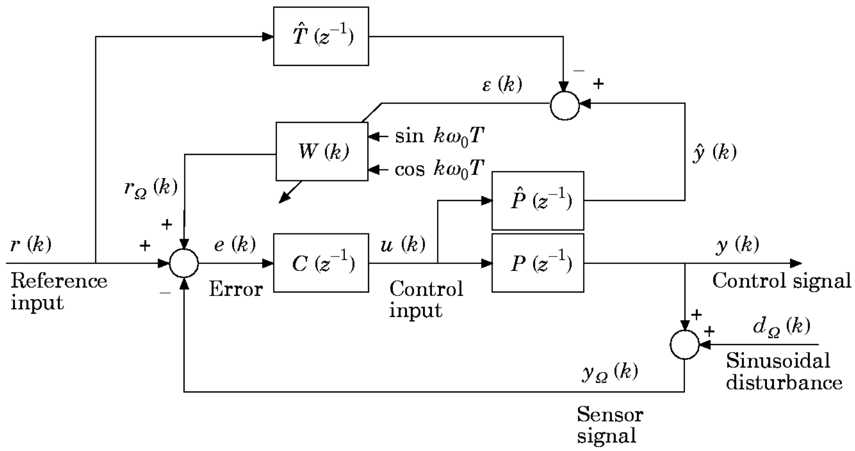

Figure 5. A block diagram of the feedforward control without altering the original closed loop configuration.

disturbance contains higher harmonic components, one can implement additional AFC at these higher frequencies. It is well known that the conventional AFC alters the original closed loop characteristics, as does the notch filter. Making a notch within or near the system bandwidth can eliminate the stability margin of the system at frequencies near ω_0 because of the associated changes in phase. Let us consider the control loop with the conventional AFC as shown in Figure 4. The cancelling signal is the same as equation (3). The conventional adaptive law governing $w_0(k)$ and $w_1(k)$ is given by the LMS algorithm as follows [10]:

$$w_0(k+1) = w_0(k) + 2\eta u(k) \sin(k\omega_0 T),$$
$$w_1(k+1) = w_1(k) + 2\eta u(k) \cos(k\omega_0 T), \tag{4}$$

where η is the step size that regulates the speed and stability of the adaptation. A linear transfer function of the AFC shown in Figure 4 can be obtained by analyzing signal propagation from the feedback error $r(k) - y_\Omega(k)$ to the AFC output $r_\Omega(k)$ by ignoring the time-varying component of the z-transform $r_\Omega(z)$ as follows [10, 15]:

$$\frac{E(z)}{R(z) - Y_\Omega(z)} = \frac{z^2 - 2z\cos\omega_0 T + 1}{z^2 - 2(1-\eta)z\cos\omega_0 T + 1 - 2\eta}, \tag{5}$$

where $E(z) = R(z) - Y_\Omega(z) - R_\Omega(z)$. Equation (5) is the transfer function of a second order digital notch filter with a notch at the disturbance frequency ω_0. The sharpness of the notch is determined by the parameter η. Therefore, a strong similarity between the notch filter and the conventional AFC exists and it may lead to instability for both cases.

3. NOVEL AFC DESIGN AND ITS ALGORITHM

A novel adaptive feed forward controller (AFC) design method, which does not alter the original closed-loop characteristics, is proposed in this section.

Compensation of a periodic disturbance can be achieved by adding a secondary synchronous signal $r_\Omega(k)$ to the reference input. The discrete system output is then decomposed as:

$$y(k) = T(z^{-1})r(k) - T(z^{-1})\{d_\Omega(k) - r_\Omega(k)\}, \tag{6}$$

where $T(z^{-1}) = \{P(z^{-1})C(z^{-1})\}/\{1 + P(z^{-1})C(z^{-1})\}$ and z^{-1} is a delay operator. Note that $T(z^{-1})$ is the closed loop transfer function. $r_\Omega(k)$ is designed to cancel the component of the system output due to $d_\Omega(k)$ by making the factor $r_\Omega(k) - d_\Omega(k) = 0$ in Equation (6) without changing the frequency response of $T(z^{-1})$. This approach is desirable because the compensation $C(z^{-1})$ can now be designed independently without considering the disturbance.

To make the factor $r_\Omega(k) - d_\Omega(k) = 0$ in Equation (6) without affecting the loop stability or the system bandwidth, the cost function J is defined as

$$\begin{aligned} J = \varepsilon^2(k) &= \{\hat{y}(k) - \hat{T}(z^{-1})r(k)\}^2 \\ &= [\hat{T}(z^{-1})\{d_\Omega(k) - r_\Omega(k)\}]^2, \end{aligned} \tag{7}$$

where $\hat{T}(z^{-1}) = \hat{P}(z^{-1})C(z^{-1})/\{1 + \hat{P}(z^{-1})C(z^{-1})\}$. Note that $\hat{T}(z^{-1})$ is the estimated transfer function. Since the output signal $\hat{y}(k)$ cannot be measured by the sensor, signal $\hat{y}(k)$ should be estimated by $\hat{y}(k) = \hat{P}(z^{-1})u(k)$. The estimate of the closed loop transfer function $T(z^{-1})$ is assumed to be perfect at the disturbance frequency ω_0; i.e., $\hat{T}(e^{-j\omega_0 T}) = T(e^{-j\omega_0 T})$. The cost function in equation (7) can be simplified by using a sinusoidal transfer function $T(e^{-j\omega_0 T})$ under steady state conditions. The modulus and phase of its transfer function are given by $Ae^{-j\Phi}$. The filtered signal of $r_\Omega(k)$, i.e., $T(z^{-1})\, r_\Omega(k)$, is represented as follows:

$$T(z^{-1})r_\Omega(k) = Aw_0(k-n)\sin(k\omega_0 T - \Phi) + Aw_1(k-n)\cos(k\omega_0 T - \Phi), \tag{8}$$

where Φ is a system delay and n is integer number of Φ/ω_0; i.e., $Int(\Phi/\omega_0)$. Substituting equation (8) into equation (7) yields the following cost function:

$$\varepsilon(k) = T(z^{-1})d_\Omega(k) - \{Aw_0(k-n)\sin(k\omega_0 T - \Phi) + Aw_1(k-n)\,cos\,(k\omega_0 T - \Phi)\} \tag{9}$$

The optimum set of Fourier coefficients, which minimizes J, may be adaptively obtained using a steepest gradient descent method. Based on the derivative of the cost function with respect to one coefficient, the update equation can be as follows:

$$w_i(k+1) = w_i(k) - \eta\frac{\partial\varepsilon^2(k)}{\partial w_i(k)} = w_i(k) - 2\eta\varepsilon(k)\frac{\partial\varepsilon(k)}{\partial w_i(k)} \qquad (i = 0, 1), \tag{10}$$

where η is a step size that determines the speed of adaptation. Substitution of equation (9) into equation (10) yields the adaptation rule as follows:

$$w_i(k+1) = w_i(k) + 2\eta\epsilon(k)\frac{\partial\{Aw_0(k-n)\sin(k\omega_0 T - \Phi) + Aw_1(k-n)\cos(k\omega_0 T - \Phi)\}}{\partial w_i(k)} \qquad (i = 0, 1). \tag{11}$$

Since the partial derivative in the above equation cannot be given in an explicit manner, based on the assumption that each w_i is the time invariant, i.e., $w_i(k - \Phi/\omega_0) = w_i(k)$ [10, 16], a modified form of the well-known LMS algorithm is produced as follows:

$$\begin{aligned} w_0(k+1) &= w_0(k) + 2\eta\varepsilon(k)A\sin(k\omega_0 T - \Phi), \\ w_1(k+1) &= w_1(k) + 2\eta\varepsilon(k)A\cos(k\omega_0 T - \Phi). \end{aligned} \tag{12}$$

The proposed AFC tracks the amplitude and phase of disturbance by driving $w_0(k) \to w_0$ and $w_1(k) \to w_1$ and, consequently, $r_\Omega(k) \to d_\Omega(k)$. The system output response due to the synchronous disturbance is cancelled at steady state. A block diagram of the proposed AFC scheme is shown in Figure 5.

The adaptive algorithm, shown in equation (12), is identical to the delayed-x LMS algorithm, which is special form of the filtered-x LMS algorithm. In a case of delayed-x LMS algorithm, Snyder, Morgan and Long derived the region of η which makes the algorithm stable for n-step delay as follows:

$$0 < \eta < \frac{1}{A^2\lambda} \sin \{\pi/(4n + 2)\}, \tag{13}$$

where n is $Int\ (\Phi/\omega_0)$ and λ is an eigenvalue of the AFC input autocorrelation matrix,

$$Expectation\begin{bmatrix} \sin^2 (k\omega_0 T) & \sin (k\omega_0 T) \cos (k\omega_0 T) \\ \cos (k\omega_0 T) \sin (k\omega_0 T) & \cos^2 (k\omega_0 T) \end{bmatrix}.$$

It should be noted that the delay n reduces the convergence region of η by a factor of $\sin \{\pi/(4n + 2)\}$. Hence, the convergence speed of an AFC algorithm is expected to decrease as the system delay Φ increases.

5. COMPUTER SIMULATIONS

A number of computer simulations were carried out in order to compare the conventional AFC and the proposed AFC.

Consider the second order integral system ($P(s) = 1/ms^2$) for computer simulations. In order to increase the system stability, the PD control is added to the feedback system. Thus, the discrete closed loop transfer function is obtained as follows [17]:

$$T(z^{-1}) = \frac{0{\cdot}198z^{-1} + 0{\cdot}198z^{-2}}{1 - 1{\cdot}044z^{-1} + 0{\cdot}0442z^{-2}},$$

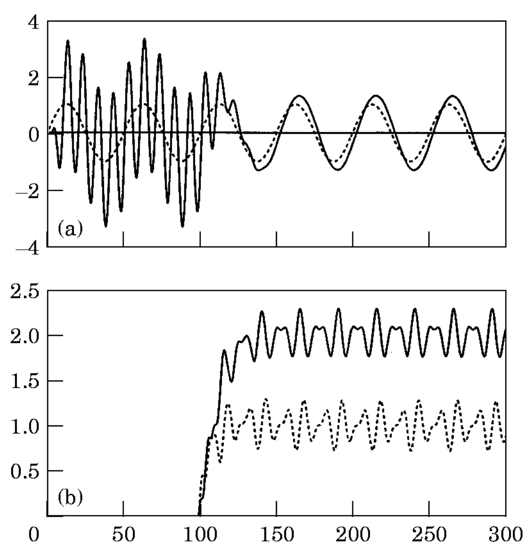

Figure 6. The response of the conventional adaptive feedforward controller ($\eta = 0{\cdot}07$). (*a*) Output response: ---, references; ———, content. (b) Time history of coefficient update: ---$\omega_0(k)$; ———, $w_1(k)$.

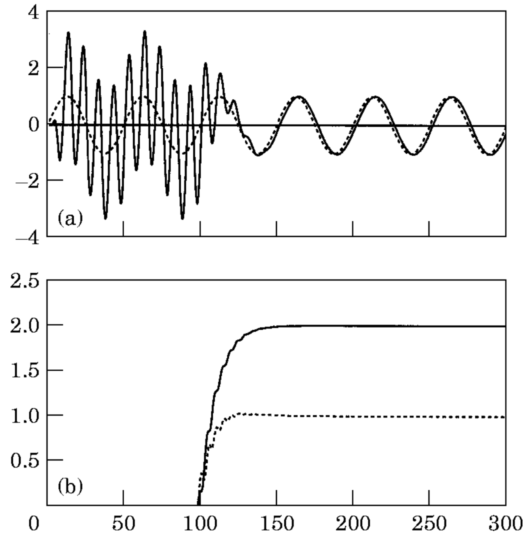

Figure 7. The response of the proposed adaptive feedforward controller ($\eta = 0{\cdot}07$). (*a*) Output response: ---, references; ———, output. (b) time history of coefficient update: ---$\omega_0(k)$; ———, $w_1(k)$.

where the sampling time is 1 s. We have performed a simulation for the non-zero reference tracking problem. The frequencies of the harmonic reference signal and the periodic disturbance were 0·01 Hz and 0·1 Hz respectively, as follows:

$$r(k) = 1{\cdot}0 \sin[2\pi(0{\cdot}01)k]$$

$$d_\Omega(k) = 1{\cdot}0 \sin[2\pi(0{\cdot}1)k] + 2{\cdot}0 \cos[2\pi(0{\cdot}1)k].$$

The initial values of the Fourier coefficients were set to zero and the convergence factor v was fixed at 0·07. The closed loop transfer function $T(z^{-1})$ was assumed to be perfectly estimated at the disturbance frequency ω_0; i.e., $\hat{T}(e^{-j\omega_0 T}) = T(e^{-j\omega_0 T})$. The resulting time history plots of the Fourier coefficients $\omega_0(k)$ and the output signal $y(k)$ are shown in Figures 6 and Figure 7. AFC compensations were enabled at the 100th step. Before AFC is enabled ($t < 100$ step), a synchronous component due to $d_\Omega(k)$ can be seen clearly in the signals $y(k)$. Within 20 steps after the compensation is on, the synchronous component in $y(k)$ is removed in both cases. In the conventional AFC case, the Fourier coefficients do not converge to constants as shown in Figure 6. Instead, they fluctuate around constant levels. By increasing the convergence factor, their fluctuation is increased. In other words, they converge on a dynamic rather than a static solution. This is in a sense forming a non-stationary least-squares solution. In the proposed AFC case, Fourier coefficients converge exactly to the steady state values of $w_0(k) = 1{\cdot}0$ and $w_1(k) = 2{\cdot}0$ after about 20 steps. This simple example illustrates better performance of the proposed AFC compared with the conventional AFC.

We also simulated the frequency response of the closed loop system with the proposed and the conventional AFC compensations. The frequency response can only be calculated by a curve fitting method after time domain simulations, since the Fourier coefficients of the conventional AFC converge on a dynamic rather than a static solution: i.e., a non-stationary process. The conventional AFC makes a notch within the system bandwidth at the vicinity of 0·1 Hz (0·628 rad/s); i.e., the sinusoidal disturbance frequency

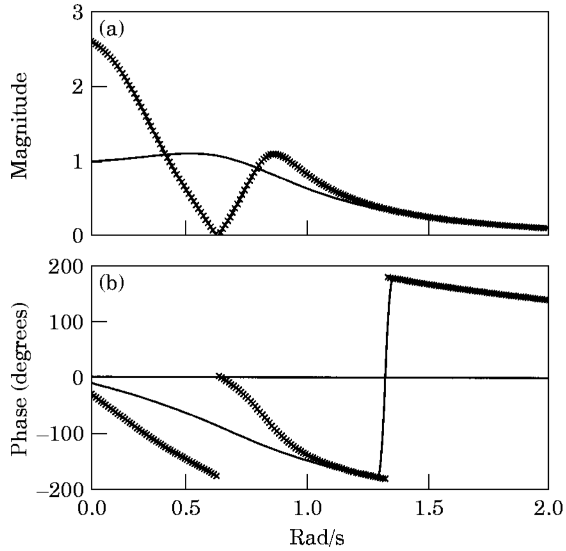

Figure 8. The transfer function of the proposed adaptive feedforward controller ($\eta = 0{\cdot}2$). (a) Magnitude plot; (b) phase plot. ———, Original closed loop transfer function $T(z^{-1})$; ×, closed loop transfer function of the conventional AFC.

ω_0 (see the line designated by crosses in Figure 8). It is clear that conventional AFC yields an effect similar to a notch filter, although it is based on an adaptive feed forward control technique. Note that the increased negative phase around the disturbance frequency may lead to instability and may limit the usefulness of this approach. With the proposed AFC, the frequency response is not altered and therefore the loop stability and the system bandwidth (see the crosses in Figure 9) are not changed at all.

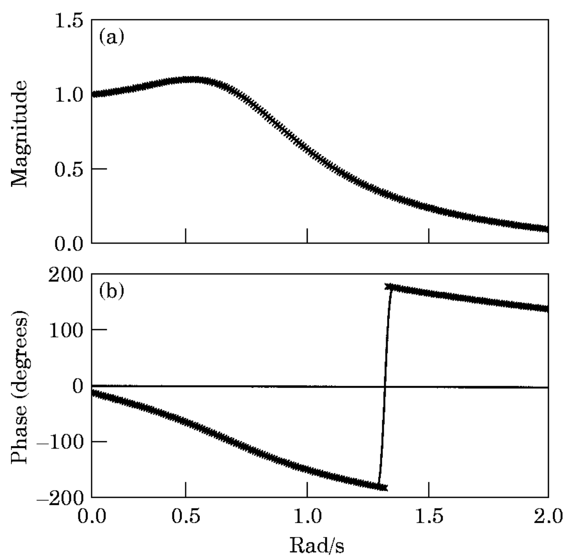

Figure 9. The transfer function of the proposed adaptive feedforward controller ($\eta = 0{\cdot}2$). (a) Magnitude plot; (b) phase plot. ———, Original closed loop transfer function $T(z^{-1})$; ×, closed loop transfer function of the proposed AFC.

6. CONCLUSIONS

A novel adaptive feedforward contoller (AFC) design method for rejecting periodic disturbance is proposed. It can be added to an existing feedback control system without altering the original closed loop configuration. The adaptive algorithm developed in this work turns out to be identical to the delayed-x LMS algorithm. The results of the simulation illustrate that the proposed AFC does not alter the original closed loop characteristics.

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